DESIGN AND ANALYSIS OF SOME ASYMMETRICAL QUALITATIVE-CUM-QUANTITATIVE EXPERIMENTS

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Introduction

Fisher (1935) was the first to tackle the problem of design and analysis of qualitative-cum-quantitative experiments. He pointed out that the assumption of additive effect of qualities and quantities (additive model) is not wholly satisfactory and proposed instead the model that quality differences may be regarded proportional to quantity applied (proportional model). Yates (1937) discussed in details the method of analysis of a 3^3 qualitative-cum-quantitative experiment in 9 plot blocks. How the knowledge of response curves helps in the choice of appropriate model for the analysis of such experiments has been discussed by Kempthorne (1951). Williams (1952) discussed a much more general proportional model, in which the proportions are not simply the quantities applied, as proposed by Fisher (1935), but are estimates which provide maximum information from the data. The appropriate method of analysis of qualitative-cum-quantitative experiments involving only non-zero levels, has been discussed by Rayner (1953) on the basis of the suggestions made by Fisher (1951). Sardana (1961) has discussed in details possible types of confounding and presented the mathematical details of analysis under the additive as well as proportional models for some of the useful experiments involving three factors.

The present paper deals with the choice of appropriate confounded design and method of analysis for the following types of commonly used 3-factor experiments involving dummies :

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n		*p*
Quantities	*Qualities*	*Quantities or qualities*
3	3	2
3	4	2
4	3	2
3	5	2

1. $3\times3\times2$ QUALITATIVE-CUM-QUANTITATIVE EXPERIMENT

Let the three factors be :

(*i*) 3 quantities of '*n*' $-n_0$, n_1, n_2, in the ratio of 0 : 1 : 2.

(*ii*) 3 qualities of '*n*' $-q_0$, q_1, q_2.

(*iii*) 2 quantities or qualities of '*p*' $-p_0$, p_1.

It would be desirable to layout experiments of this type in 6 plot blocks. Sardana (1961) has pointed out that in such experiments laid out in 6 plot blocks, the quality main effect cannot be kept free from confounding. Assuming that the experimenter is prepared to sacrifice some information on the quality main effect, the choice of the appropriate design and the method of analysis of such experiments can be examined.

For a $3\times3\times2$ factorial experiment involving only quantitative factors, Yates (1935) has given a balanced confounded design in 6 plot blocks involving 4 replications confounding *NQ* and *NQP*. An alternative $3\times3\times2$ balanced design can be obtained in 6 plot blocks by first getting a 3×3 balanced design in 3 plot blocks confounding the two factor interaction in two replications. The required $3\times3\times2$ balanced design in 6 plot blocks involving 2 replications is then obtained by combining each treatment combination of the factors '*n*' and '*q*' in each block of the balanced 3×3 design with the two levels of the factor '*p*'.

Designs for $3\times3\times2$ qualitative-cum-quantitative experiments in 6 plot blocks can be derived from either of the above mentioned designs by using dummy treatments, where necessary. The design derived from Yates's design confounding *NQ* (*I*) and *NQ* (*I*)*P* in two replications is called design I, while the other derived from the above mentioned alternative method is called design II. These designs are given in Appendix I. In the analysis of the designs presented in this paper, the sum of the fitted constants has in all cases been assumed to be a constant quantity and not zero.

1·1. Analysis of 3×3×2 qualitative-cum-quantitative experiment

1·1·1. *Design I.* It can be shown that in this design, besides NQ and NQP ; Q and QP are affected by block differences. The following scheme of constants has been used in writing the estimates corresponding to the confounded effects :

Confounded effect	*D.F.*	*Constants*
Q	2	t_0, t_1, t_2
NQ	2	u_0, u_1, u_2
QP	2	v_0, v_1, v_2
NQP	2	w_0, w_1, w_2

Using the additive model, the normal equations of the confounded effects Q and QP, written after eliminating the block effects, simplify as under :

Normal equation of Q

$$\left.\begin{aligned} \tfrac{46}{6}t_0-\tfrac{1}{3}(u_1-u_2)&=T_0-\tfrac{1}{6}(2B_{11}+B_{21}+B_{31}+2B_{12}+B_{22}+B_{32}) \\ \tfrac{46}{6}t_1-\tfrac{1}{3}(u_2-u_0)&=T_1-\tfrac{1}{6}(B_{11}+B_{21}+2B_{31}+B_{12}+B_{22}+2B_{32}) \\ \tfrac{46}{6}t_2-\tfrac{1}{3}(u_0-u_1)&=T_2-\tfrac{1}{6}(B_{11}+2B_{21}+B_{31}+B_{12}+2B_{22}+B_{32}) \end{aligned}\right\} \quad \ldots(1)$$

Normal equation of QP

$$\left.\begin{aligned} 7v_0-(w_1-w_2)&=V_0-\tfrac{1}{6}(-B_{21}+B_{31}+B_{22}-B_{32}) \\ 7v_1-(w_2-w_0)&=V_1-\tfrac{1}{6}(-B_{11}+B_{21}+B_{12}-B_{22}) \\ 7v_2-(w_0-w_1)&=V_2-\tfrac{1}{6}(\ B_{11}-B_{31}+B_{32}-B_{12}) \end{aligned}\right\} \quad \ldots(2)$$

Where B_{ml}=Total of m-th block of l-th replication ;
$m=1, 2, 3$ and $l=1, 2$

T_j=Total of all the observations which receive 'n' either at the first or at the second level through q_j ; for $j=0$, 1 and 2.

V_j=Total of all the observations involving 'p' at second level and 'n' either at first or second level supplied through q_j minus the total of all the observations involving 'p' at first level and 'n' either at first or second level supplied through q_j.

The set of equations (1) indicates that uuder the additive model in the design Q and NQ are not estimable mutually independently. Also set (2) indicates that QP and NQP cannot be estimated mutually independently.

Similarly, under the pro ortional model the normal equations are mixed up in the same way *i.e.* Q and NQ; QP and NQP.

Thus in design I, we can either get joint estimates of Q, NQ; and QP, NQP following Fisher's technique (1935) or estimate Q and QP assuming the effects NQ and NQP to be negligible.

1·1·2. *Design II.* It can be shown that in this design, both under the additive and proportional models, the confounded effects are Q and NQ.

The normal equations of Q and NQ, under the additive model, provide the following solutions :

$$\hat{t}_j = \tfrac{3}{20}\, T_j' \text{ for } j=0,1,2$$

$$\hat{u}_j = \tfrac{1}{4}\, U_j' \text{ for } j=0,1,2$$

where
$$T_j' = T_j - \tfrac{2}{6}\,[B]t_j$$
$$U_j' = U_j - \tfrac{2}{6}\,[B]u_j$$

$[B]t_j$ = sum of those block totals which centain the treatment combinations involving either n_1q_j or n_2q_j.

$[B]u_j$ = Sum of those block totals which contain the treatment combinations involving n_2q_j minus the sum of those block totals whieh contain the treatment combinations involving n_1q_j

The sum of squares due to Q and NQ are

$$\left\{ \sum_{j=0}^{2} \hat{t}_j T_j' - \tfrac{1}{3}\left(\sum_{j=0}^{2} \hat{t}_j\right)\left(\sum_{j=0}^{2} T_j'\right)\right\}$$

and
$$\left\{ \sum_{j=0}^{2} \hat{u}_j U_j' - \tfrac{1}{3}\left(\sum_{j=0}^{2} \hat{u}_j\right)\left(\sum_{j=0}^{2} u_j'\right)\right\} \text{ respectively.}$$

The relative loss of information on each degree of freedom of Q is $\frac{1}{6}$, while on each degree of freedom of NQ is $\frac{1}{2}$. Thus the total relative loss of information is 4/3.

The normal equations for Q and NQ, under the proportional model, simplify as under :—

Normal equation of Q

$$16t_j - 2u_j = T_j'' - \tfrac{2}{6}[B]'t_j \tag{3}$$

Normal equation of NQ

$$\tfrac{64}{6}u_j - 2t_j = u_j'' - \tfrac{2}{6}[B]'u_j \tag{4}$$

Where T_j''=Total of all the observations which involve the combination n_1q_j plus twice the total of all the observations which involve the combination n_2q_j.

$[B]'t_j$=sum of those block totals which contain the treatment combinations involving n_1q_j plus twice the sum of those block totals which contain the treatment combinations involving n_2q_j.

U_j''=Total of all the observations which involve the combination n_2q_j minus twice the total of all the observations which involve the combination n_1q_j.

$[B]'u_j$=sum of those block totals which contain the treatment combinations involving n_2q_j minus twice the sum of those block totals which contain the treatment combinations involving n_1q_j.

It is evident from equations (3) and (4) that the effects Q and NQ cannot be estimated mutually independently. Thus one of the effects can be estimated by assuming the other to be negligible. Obviously in such situations one would prefer to estimate Q instead of NQ. Assuming NQ to be negligible, estimates of Q are given by :—

$$\hat{t}_j = T_j''' = T_j'' - \tfrac{2}{6}\,[B]'t_j \qquad \text{for} \qquad j=0,\ 1 \text{ and } 2.$$

The sum of squares due to Q (2d.f.) is given by

$$\left\{ \sum_{j=0}^{2} \hat{t}_j\, T_j''' - \tfrac{1}{3}\left(\sum_{j=0}^{2} \hat{t}_j\right)\left(\sum_{j=0}^{2} T_j'''\right)\right\}.$$

The relative loss of information on each degree of freedom of Q is 1/5.

2. 3×4×2 QUALITATIVE-CUM-QUANTITATIVE EXPERIMENTS

Let the three factors be :

(*i*) 3 quantities of 'n'—n_0, n_1, n_2 in the ratio 0 : 1 : 2.

(*ii*) 4 qualities of 'n'—q_0, q_1, q_2 and q_3

(*iii*) 2 quantities or qualities of 'p'—p_0, p_1.

An experiment of this type would usually be laid out in blocks of size smaller than 24. Sardana (1961) has discussed the confounded design for such experiments in 12 plot blocks. We shall now discuss confounded designs for such experiments in 6 plot blocks.

Design for $3\times4\times2$ qualitative-cum-quantitative experiment in 6 plot blocks can be derived from the design for a corresponding factorial experiment involving only quantitative factors, by using dummy treatments where necessary. To obtain the required design for a $3\times4\times2$ factorial experiment involving only quantitative factors, we first obtain a 4×4 balanced design in 4 plot blocks, confounding the two factor interaction in 3 replications. In all the blocks of this design, the treatment combinations involving the fourth level of the first factor is omitted to provide a balanced design for 3×4 in 3 plot blocks involving 3 replications. The treatment combinations of the factors 'n' and 'q' in all the blocks of this 3×4 balanced design are then combined with the two levels of the factor 'p' to give rise to a balanced $3\times4\times2$ design in 6 plot blocks involving 3 replications. Design for the qualitative-cum-quantitative experiment derived from this is given in Appendix II.

2·1. Analysis of $3\times4\times2$ qualitative-cum-quantitative experiment

It can be shown that, both under the additive and proportional models, the confounded effects are Q and NQ. The following schemes of constants have been used in writing the estimates corresponding to the confounded effects :

Confounded effect	*D.F.*	*Constants*
Q	3	t_0, t_1, t_2, t_3
NQ	3	u_0, u_1, u_2, u_3

The normal equations of Q and NQ, under the additive model, provide the following solutions

$$\hat{t}_j=\tfrac{3}{28}\,\{T_j-\tfrac{2}{6}[B]t_j\} \qquad \text{for} \qquad i=0,\,1,\,2,\,3$$

$$\hat{u}_j=\tfrac{3}{20}\,\{U_j-\tfrac{2}{6}[B]u_j\} \qquad \text{for} \qquad j=0,\,1,\,2,\,3.$$

Where T_j=Total of all the observations which receive 'n' either at the first or at the second level through q_j.

$[B]t_j$=Sum of those block totals which contain the treatment combinations involving either n_1q_j or n_2q_j.

U_j=Total of all the observations which involve the combination n_2q_j minus the total of all the observations which involve the combination n_1q_j.

$[B]u_j$ = Sum of those block totals which contain the treatment combinations involving n_2q_j minus the sum of those block totals which contain the treatment combinations involving n_1q_j.

The sum of squares due to Q and NQ are

$$\left\{\sum_{j=0}^{3} \hat{t}_j\, T_j' - \tfrac{1}{4}\left(\sum_{j=0}^{3} \hat{t}_j\right)\left(\sum_{j=0}^{3} T_j'\right)\right\}$$

and

$$\left\{\sum_{j=0}^{3} \hat{u}_j\, u_j' - \tfrac{1}{4}\left(\sum_{j=0}^{3} \hat{u}_j\right)\left(\sum_{j=0}^{3} u_j'\right)\right\} \qquad \text{respectively,}$$

where

$$T_j' = T_j - \tfrac{2}{6}[B]t_j$$
$$u_j' = u_j - \tfrac{2}{6}[B]u_j.$$

The relative loss of information on each d.f. of Q and NQ is $\frac{2}{9}$ and $\frac{4}{9}$ respectively. Thus the total relative loss of information is 2.

The normal equations for Q and NQ, under the proportional model, simplify as under :

Normal equation for Q is

$$\tfrac{136}{6}t_j + 2u_j = T_j'' - \tfrac{2}{6}\,[B]'t_j \qquad (5)$$

Normal equation for NQ is

$$\tfrac{104}{4}u_j + 2t_j = U_j'' - \tfrac{2}{6}\,[B]'u_j \qquad (6)$$

Where T_j'' = Total of all the observations which involve the combination n_1q_j plus twice the total of all the observations which involve the combination n_2q_j.

$[B]'t_j$ = Sum of those block totals which contain the treatment combinations involving n_1q_j plus twice the sum of those block totals which contain the treatment combinations involving n_2q_j.

U_j'' = Total of all the observations which involve the combination n_2q_j minus twice the total of all the observations which involve the combination n_1q_j.

$[B]'u_j$ = Sum of those block totals which contain the treatment combination involving n_2q_j minus twice the sum of those block totals which contain the treatment combination involving n_1q_j.

Equations (5) and (6) indicate that the effects Q and NQ cannot be estimated mutually independently. Obviously in such situations one would prefer to estimate Q instead of NQ. Assuming NQ to be negligible, estimates of Q are given by :

$$\hat{t}_j = \frac{3}{68} T_j''' = \frac{3}{68} [T_j'' - \tfrac{2}{6}[B]'t_j].$$

The sum of squares due to $Q(3d.f.)$ is given by

$$\left\{ \sum_{j=0}^{3} \hat{t}_j T_j''' - \tfrac{1}{4} \left(\sum_{j=0}^{3} \hat{t}_j \right) \left(\sum_{j=0}^{3} T_j''' \right) \right\}.$$

The relative loss of information on each $d.f.$ of Q is $\frac{11}{45}$.

3. $4 \times 3 \times 2$ QUALITATIVE-CUM-QUANTITATIVE EXPERIMENT

Let the three factors be :

(*i*) 4 quantities of 'n' $-n_0, n_1, n_2, n_3$ in the ratio of 0 : 1 : 2 : 3

(*ii*) 3 qualities of 'n' $-q_0, q_1, q_2$

(*iii*) 2 quantities or qualities of 'p' $-p_0, p_1$.

In section 2, the procedure for constructing a confounded design in 6 plot blocks involving 3 replications for a $3 \times 4 \times 2$ factorial experiment involving only quantitative factors has been indicated. If the factors at 3, 4 and 2 levels are taken as 'q', 'n' and 'p' respectively, then the required confounded design for $4 \times 3 \times 2$ qualitative-cum-quantitative experiment can be obtained by using dummy treatments, where necessary. The design so derived is given in Appendix III.

3·1. Analysis of $4 \times 3 \times 2$ qualitative-cum-quantitative experiment

It can be shown that, both under the additive and proportional models, the confounded effects are N, Q and NQ. The following scheme of constants has been used in writing the estimates corresponding to the confounded effects under the additive model :

Confounded effect	*D.F.*	*Constants*
N	3	t_0, t_1, t_2, t_3
Q	2	u_0, u_1, u_2
$NQ(I)$	2	v_0, v_1, v_2
$NQ(J)$	2	w_0, w_1, w_2.

The normal equations of the above effects under the additive model provide the following solutions :

$$\hat{t}_i=\tfrac{1}{16}\{T_i-\tfrac{2}{6}[B]t_i\} \quad \text{for} \quad i=0, 1, 2, 3$$

$$\hat{u}_j=\tfrac{1}{16}\{U_j-\tfrac{2}{6}[B]u_j\} \quad \text{for} \quad j=0, 1, 2$$

$$\hat{v}_j=\tfrac{1}{10}\{V_j-\tfrac{2}{6}[B]v_j\} \quad \text{for} \quad j=0, 1, 2$$

$$\hat{w}_j=\tfrac{1}{10}\{W_j-\tfrac{2}{6}[B]w_j\} \quad \text{for} \quad j=0, 1, 2$$

Where T_i=Total of all the observations which involve the n_i level of the factor 'n'.

$[B]t_i$=Sum of all those block totals which contain the treatment combinations involving n_i level of the factor 'n'.

U_j=Sum of all the observations which involve n_1, or n_2 or n_3 level of 'n' supplied through q_j.

$[B]u_j$=Sum of all those block totals which contain the treatment combinations involving n_1q_j or n_2q_j or n_3q_j.

V_j=Sum of all the observations which involve n_1q_j or $n_2q_{(j+1)}$ or $n_3q_{(j+2)}$; the suffixes $(j+1)$ and $(j+2)$ are taken with mod (3).

$[B]v_j$=Sum of all those block totals which contain the treatment combinations involving n_1q_j or $n_2q_{(j+2)}$ or $n_3q_{(j+2)}$

W_j=Sum of all the observations which involve $n_1q_{(j+2)}$ or $n_2q_{(j+1)}$ or n_3q_j.

$[B]w_j$=Sum of all those block totals which contain the treatment combinations involving $n_1q_{(j+2)}$ or $n_2q_{(j+1)}$ or n_3q_j.

The sum of squares due to N, Q, $NQ(I)$ and $NQ(J)$ are given by :

$$\left\{\sum_{i=0}^{3}\hat{t}_i T_i'-\tfrac{1}{4}\left(\sum_{i=0}^{3}\hat{t}_i\right)\left(\sum_{i=0}^{3}T_i'\right)\right\},$$

$$\left\{\sum_{j=0}^{2}\hat{u}_j U_j'-\tfrac{1}{3}\left(\sum_{j=0}^{2}\hat{U}j\right)\left(\sum_{j=0}^{2}U_j'\right)\right\},$$

$$\left\{\sum_{j=0}^{2}\hat{v}_j V_j'-\tfrac{1}{3}\left(\sum_{j=0}^{2}\hat{v}_j\right)\left(\sum_{j=0}^{2}V_j'\right)\right\}$$

and $$\left\{\sum_{j=0}^{2}\hat{w}_j W_j'-\tfrac{1}{3}\left(\sum_{j=0}^{2}\hat{w}_j\right)\left(\sum_{j=0}^{2}W_j'\right)\right\}$$

respectively, where

$$T_i' = T_i - \tfrac{2}{6}[B]t_i$$
$$U_j' = U_j - \tfrac{2}{6}[B]u_j$$
$$V_j' = V_j - \tfrac{2}{6}[B]v_j$$
$$W_j' = W_j - \tfrac{2}{6}[B]w_j.$$

The relative loss of information on each degree of freedom of N, Q, $NQ(I)$ and $NQ(J)$ is $\frac{1}{9}$, $\frac{1}{9}$, $\frac{4}{9}$ and $\frac{4}{9}$ respectively. Thus, the total relative loss of information is 7/3.

Since in experiments of this type three non zero levels are being tried the effects Q, NQ, QP and NQP under the proportional model are defined as follows :

$Q(2\ d.f.)$ is defined as the two independent comparisons between U_0'', U_1'', U_2'', where

U_j'' = Sum of all the observations involving n_1q_j plus twice the sum of all the observations involving n_2q_j plus thrice the sum of all the observations involving n_3q_j.

The 4 degrees of freedom for NQ are divided into two components each carrying 2 degrees of freedom *viz.* (*i*) "quality × quantity" used to test the model and these are the two independent comparisons between V_0'', V_1'', V_2'', (*ii*) "quality × quadratic effect of quantity" obtained as two independent comparisons between W_0'', W_1'', W_2'' where

V_j'' = Sum of all the observations involving n_2q_j plus four times the sum of all the observations involving n_1q_j minus twice the sum of all the observations involving n_3q_j.

W_j'' = Sum of all the observations involving n_1q_j plus the sum of all the observations involving n_3q_j minus twice the sum of all the observations involving n_2q_j.

$QP(2\ d.f.)$ is defined as the interaction of the 3×2 table between 'q' and 'p', summed over $(n_1+2n_2+3n_3)$. The NQP interaction is divided into two components each carrying 2 degrees of freedom *viz.*, (*i*) "$QP\times$quantity" which is obtained as the interaction of the 3×2 table between 'q' and 'p' summed over $(4n_1+n_2-2n_3)$, and (*ii*) "$QP\times$quadratic effect of quantity", which is obtained as the interaction of the 3×2 table between 'q' and 'p' summed over $(n_1-2n_2+n_3)$.

The following scheme of constants is used in writing the estimates corresponding to the confounded effects under the proportional model :

Confounded Effect	*D.F.*	*Constants*
N	3	t_0, t_1, t_2, t_3
Q	2	u_0', u_1', u_2'
Quality × quantity	2	v_0', v_1', v_2'
Quality × quadratic effect of quantity	2	w_0', w_1', w_2'

The normal equations of the above effects simplify as under :

$$\hat{t}_i = \frac{T_i'}{16} \qquad \text{for } i=0, 1, 2, 3,$$

where T_i' has already been defined :

Normal equation for quality main effect

$$\tfrac{424}{6}\, u_j' + \tfrac{72}{6}\, v_j' = U_j''' \qquad \text{for } j=0, 1, 2, \tag{7}$$

Normal equation for 'quality × quantity'

$$\tfrac{456}{6}\, v_j' + \tfrac{72}{6}\, u_j' = V_j''' \qquad \text{for } j=0, 1, 2, \tag{8}$$

$$\hat{w}_j' = \tfrac{1}{24}\, W_j'''$$

where

$$U_j''' = U_j'' - \tfrac{2}{6}\, [B]u_j'$$

$$V_j''' = V_j'' - \tfrac{2}{6}\, [B]v_j'$$

$$W_j''' = W_j'' - \tfrac{2}{6}\, [B]w_j'$$

$[B]u_j'$ = Sum of those block totals which contain observations involving n_1q_j plus twice the sum of those block totals which contain observations involving n_2q_j plus thrice the sum of those block totals which contain observations involving n_3q_j.

$[B]v_j'$ = Sum of those block totals which contain observations involving n_2q_j plus four times the sum of those block totals which contain observations involving n_1q_j minus twice the sum of those block totals which contain observations involving n_3q_j.

$[B]w_j'$ = Sum of those block totals which contain observations involving n_1q_j plus the sum of those block totals which contain the observations involving n_3q_j minus twice the sum of those block totals which contain observations involving n_2q_j.

Equations (7) and (8) indicate that the 'quality' main effect and 'quality × quantity' interaction cannot be estimated mutually independently. Obviously in such situations one would prefer to estimate the 'quality' main effect. Assuming 'quality × quantity' interaction to be negligible, estimates of Q are given by :

$$\hat{u}_j' = \frac{6}{424} U_j'''.$$

The sum of squares due to N, Q, and 'quality × quadratic effect of quantity' are given by

$$\left\{ \sum_{i=0}^{3} \hat{t}_i T_i' - \tfrac{1}{4}\left(\sum_{i=0}^{3} \hat{t}_i \right)\left(\sum_{i=0}^{3} T_i' \right) \right\},$$

$$\left\{ \sum_{j=0}^{2} \hat{u}_j' U_j''' - \tfrac{1}{3}\left(\sum_{j=0}^{2} \hat{u}_j' \right)\left(\sum_{j=0}^{2} U_j''' \right) \right\}$$

and

$$\left\{ \sum_{j=0}^{2} \hat{w}_j' W_j''' - \tfrac{1}{3}\left(\sum_{j=0}^{2} \hat{w}_j' \right)\left(\sum_{j=0}^{2} W_j''' \right) \right\} \text{respectively.}$$

The relative loss of information on each degree of freedom of N, Q, and 'quality × quadratic effect of quantity' is $\frac{1}{9}$, $\frac{10}{63}$ and $\frac{1}{3}$ respectively.

4. 3×5×2 QUALITATIVE-CUM-QUANTITATIVE EXPERIMENTS

Let the three factors be

(*i*) 3 quantities of 'n'—n_0, n_1, n_2 in the ratio of 0 : 1 : 2

(*ii*) 5 qualities of 'n'—q_0, q_1, q_2, q_3, q_4

(*iii*) 2 quantities or qualities of 'p'—p_0, p_1.

An experiment of this type would usually be laid out in blocks of size smaller than 30. In this section we shall discuss the confounded design for such experiments in 6 plot blocks.

Kishen and Srivastava (1959) have given a confounded design in 6 plot blocks involving 4 replications for a 3×5×2 factorial experiment involving only quantitative factors. The required design for a corresponding qualitative-cum-quantitative experiment can be obtained by using dummy treatment where necessary The design so obtained is called design I and is given in Appendix IV.

An alternative design for such experiments can be obtained by first getting a 5×5 balanced design in 5 plot bocks involving 4 replications confounding the two factor interaction. In all the blocks of the design, the treatment combinations involving either the fourth or fifth level of the first factor are omitted to provide a balanced design for 3×5 in 3 plot blocks confounding Q and NQ in 4 replications. The treatment combinations of the factors 'n' and 'q' in all the blocks of the 3×5 balanced design are then combined with the two levels of the factor 'p' to give rise to a balanced $3 \times 5 \times 2$ design in 6 plot blocks in 4 replications. Design for a corresponding qualitative-cum-quantitative experiment can be obtained by using dummy treatments where necessary. The design so derived is called design II and is given in Appendix IV.

4·1. Analysis of $3 \times 5 \times 2$ qualitative-cum-quantitative experiment

(*a*) *Design I.* It can be shown that in the design, both under the additive and proportional models, the confounded effects are Q, NQ, QP, and NQP. The following scheme of constants is used in writing the estimates corresponding to the confounded effects :

Confounded effect	*D.F.*	*Constants*
Q	4	t_0, t_1, t_2, t_3, t_4
NQ	4	u_0, u_1, u_2, u_3, u_4
QP	4	v_0, v_1, v_2, v_3, v_4
NQP	4	w_0, w_1, w_2, w_3, w_4.

Using the additive model, the normal equations of the confounded effects simplify as under :

Normal equation of Q

$$\tfrac{82}{6}\, t_j + \tfrac{10}{6}\, w_j = T_j' \text{ for } j=0, 1, 2, 3, 4 \tag{9}$$

Normal equation of NQ

$$\tfrac{86}{6}\, u_j - \tfrac{10}{6}\, v_j = U_j' \text{ for } j=0, 1, 2, 3, 4 \tag{10}$$

Normal equation of QP

$$\tfrac{86}{6}\, v_j - \tfrac{10}{6}\, u_j = V_j' \text{ for } j=0, 1, 2, 3, 4 \tag{11}$$

Normal equation of NQP

$$\tfrac{66}{6}\, w_j + \tfrac{10}{6}\, t_j = W_j' \text{ for } j=0, 1, 2, 3, 4 \tag{12}$$

Where $T_j' = T_j - \tfrac{1}{6}\, [B]t_j$

T_j = Sum of those observations which involve 'n_1', or 'n_2' level of 'n' supplied through q_j.

$[B]t_j$=Sum of those block totals which contain the treatment combinations involving n_1q_j plus the sum of those block totals which contain the treatment combinations involving n_2q_j.

$U_j' = U_j - \frac{1}{6}[B]u_j$.

U_j=Sum of those observations which involve 'n_2q_j' minus the sum of those observations which involve n_1q_j.

$[B]u_j$=Sum of those block totals which contain the treatment combinations involving n_2q_j minus the sum of those block totals which contain the treatment combinations involving n_1q_j.

$V_j' = V_j - \frac{1}{6}[B]v_j$.

V_j=Sum of those observations which involve $n_1q_jp_1$ or $n_2q_jp_1$, minus the sum of those observations which involve $n_1q_jp_0$ or $n_2q_jp_0$.

$[B]v_j$=Sum of those block totals which contain the treatment combination $n_1q_jp_1$ plus the sum of those block totals which contain the treatment combination $n_2q_jp_1$, minus the sum of those block totals which contain the treatment combination $n_1q_jp_0$ minus the sum of those block totals which contain the treatment combination $n_2q_jp_0$.

$W_j' = W_j - \frac{1}{6}[B]w_j$.

W_j=Sum of those observations which involve $n_2q_jp_1$ or $n_1q_jp_0$ minus the sum of those observations which involve $n_2q_jp_0$ or $n_1q_jp_1$.

$[B]w_j$=Sum of those block totals which contain the treatment combination $n_2q_jp_1$, plus the sum of those block totals which contain the treatment combination $n_1q_jp_0$ minus the sum of those block totals which contain the treatment combination $n_2q_jp_0$ minus the sum of those block totals which contain the treatment combination $n_1q_jp_1$.

Equations (9) and (12) show that the effects Q and NQP cannot be estimated mutually independently. Further, equations (10) and (11) show that NQ and QP cannot be estimated mutually independently. Thus one of the effects Q or NQP can be estimated assuming the other to be negligible and similarly one of the effects NQ or NQP can be estimated assuming the other to be negligible.

Under the proportional model, the normal equations of the confounded effects simplify as under :

Normal equation for Q

$$\frac{206}{6} t_j + \frac{3}{6} u_j + \frac{25}{6} w_j = T_j'' \tag{13}$$

Normal equation for NQ

$$\frac{214}{6} u_j + \frac{3}{5} t_j - \frac{25}{6} v_j = U_j'' \tag{14}$$

Normal equation for QP

$$\frac{210}{6} v_j - \frac{25}{6} u_j - \frac{15}{6} w_j = V_j'' \tag{15}$$

Normal equation for NQP

$$\frac{170}{6} w_j + \frac{25}{6} t_j - \frac{15}{6} v_j = W_j'' \tag{16}$$

where T_j'', U_j'', V_j'' and W_j'' can be defined with due care to the constants to be assigned to different treatment combinations.

Equations (13) to (15) show that the confounded effects Q, NQ QP and NQP cannot be estimated mutually independently. Effects Q and QP can be estimated assuming NQ and NQP to be negligible or vice-versa.

(*b*) *Design II.* It can be shown that, both under the additive and proportional models, the confounded effects are Q and NQ. Using the same scheme of constants as in case of design I, the normal equations of the confounded effects, under the additive model, provide the following solutions :

$$\hat{t}_j = \frac{1}{12} \left[T_j - \tfrac{2}{6}[B]'t_j\right]$$

$$\hat{u}_j = \frac{3}{28} \left[U_j - \tfrac{2}{6}[B]'u_j\right]$$

Where T_j and U_j have already been defined, and

$[B]'t_j$ = Sum of those block totals which contain the treatment combinations involving either n_1q_j or n_2q_j.

$[B]'u_j$ = Sum of those block totals which contain the treatment combinations involving n_2q_j minus the sum of those block totals which contain the treatment combination involving n_1q_j.

The sum of squares due to Q and NQ are ;

$$\left\{\sum_{j=0}^{4} \hat{t}_j T_j' - \tfrac{1}{5}\left(\sum_{j=0}^{4} \hat{t}_j\right)\left(\sum_{j=0}^{4} T_j'\right)\right\} \text{ and}$$

$$\left\{\sum_{j=0}^{4} \hat{u}_j u_j' - \tfrac{1}{5}\left(\sum_{j=0}^{4} \hat{u}_j\right)\left(\sum_{j=0}^{4} U_j'\right)\right\} \text{ respectively, where}$$

$$T_j' = T_j - \tfrac{2}{6}[B]'t_j$$
$$U_j' = U_j - \tfrac{2}{6}[B]'u_j.$$

The relative loss of information on each degree of freedom of Q and NQ is $\frac{1}{4}$ and $\frac{5}{12}$ respectively. Thus the total relative loss of information is $\frac{8}{3}$.

The normal equations of the confounded effects, under the proportional model, simplify as under :

Normal equation of Q

$$\frac{88}{3}\, t_j - 2u_j = T_j''' \tag{17}$$

Normal equation of NQ

$$24u_j - 2t_j = U_j''' \tag{18}$$

Where T_j''' = Sum of those observations which involve n_1q_j plus twice the sum of those observations which involve n_2q_j minus $\frac{2}{6}$ $[B]''t_j$.

$[B]''t_j$ = Sum of those block totals which contain the treatment combination involving n_1q_j plus twice the sum of those block totals which contain the treatment combination involving n_2q_j.

U_j''' = Sum of those observations which involve n_2q_j minus twice the sum of those observations which involve n_1q_j minus $\frac{2}{6}[B]''u_j$.

$[B]''u_j$ = Sum of those block totals which contain the treatment combination involving n_2q_j minus twice the sum of those block totals which contain the treatment combination involving n_1q_j.

The equations (17) and (18) indicate that the effects Q and NQ cannot be estimated mutually independently. Obviously in such situations one would prefer to estimate Q instead of NQ. Assuming NQ to be negligible, estimates of Q are given by :

$$\hat{t}_j = \frac{3}{88} T_j'''$$

The sum of squares due to Q is given by

$$\left\{ \sum_{j=0}^{4} \hat{t}_j\, T_j''' - \frac{1}{5}\left(\sum_{j=0}^{4} \hat{t}_j\right)\left(\sum_{j=0}^{4} T_j'''\right) \right\}.$$

The relative loss of information on each degree of freedom of Q is $\frac{4}{15}$.

5. Summary

The paper deals with the choice of appropriate confounded design and the method of analysis, both under the additive and proportional models, for the following types of commonly used 3-factor qualitative-cum-quantitative experiments involving dummies:

'n'		'p'
Quantities	*Qualities*	*Quantities or qualities*
3	3	2
3	4	2
4	3	2
3	5	2

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APPENDIX I

Designs for 3×3×2 qualitative-cum-quantitative experiments in 6 plot blocks

DESIGN I

Replication 1			*Replication 2*		
Block 1	*Block 2*	*Block 3*	*Block 1*	*Block 2*	*Block 3*
nqp	*nqp*	*nqp*	*nqp*	*nqp*	*nqp*
100	200	0-0	200	0-0	100
210	0-0	110	0-0	110	210
0-0	120	220	120	220	0-0
201	0-1	101	101	201	0-1
0-1	111	211	211	0-1	111
121	221	0-1	0-1	121	221

DESIGN II

Replication 1			*Replication 2*		
Block 1	*Block 2*	*Block 3*	*Block 1*	*Block 2*	*Block 3*
nqp	*nqp*	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1	0-1	0-1
120	100	110	110	120	100
121	101	111	111	121	101
210	220	200	220	200	210
211	221	201	221	201	211

APPENDIX II

Design for 3×4×2 qualitative-cum-quantitative experiment in 6 plot blocks

Replication I

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1
100	110	130	120
101	111	131	121
220	230	210	200
221	231	211	201

Replication II

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1
120	110	100	130
121	111	101	131
210	220	230	200
211	221	231	201

Replication III

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1
110	130	120	100
111	131	121	101
200	220	230	210
201	221	231	211

APPENDIX III

Design for 4×3×2 qualitative-cum-quantitative experiment in 6 plot blocks

Replication I

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	0-0	120	0-0
0-1	0-1	121	0-1
100	110	200	210
101	111	201	211
220	320	310	300
221	321	311	301

Replication II

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	110	0-0	0-0
0-1	111	0-1	0-1
120	220	200	100
121	221	201	101
210	300	320	310
211	301	321	311

Replication III

Block 1	Block 2	Block 3	Block 4
nqp	*nqp*	*nqp*	*nqp*
0-0	0-0	100	0-0
0-1	0-1	101	0-1
110	220	210	120
111	221	211	121
200	310	320	300
201	311	321	301

APPENDIX IV

Designs for 3×5×2 qualitative-cum-quantitative experiments in 6 plot blocks

Design 1

Replication 1

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
201	211	221	231	241
131	141	101	111	121
230	240	200	210	220
140	100	110	120	130
0-1	0-1	0-1	0-1	0-1

Replication 2

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
201	231	211	241	221
141	121	101	131	111
240	220	200	230	210
120	100	130	110	140
0-0	0-1	0-1	0-1	0-1

Replication 3

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
201	221	241	211	231
111	131	100	121	141
210	230	200	220	240
130	100	120	140	110
0-1	0-1	0-1	0-1	0-1

Replication 4

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
201	241	231	221	211
121	111	101	141	131
220	210	200	240	230
110	100	140	130	120
0-1	0-1	0-1	0-1	0-1

Design II

Replication 1

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1	0-1
140	100	110	120	130
141	101	111	121	131
230	240	200	210	220
231	241	201	211	221

Replication 2

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1	0-1
120	130	140	100	110
121	131	141	101	111
240	200	210	220	230
241	201	211	221	231

Replication 3

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1	0-1
130	140	100	110	120
131	141	101	111	121
210	220	230	240	200
211	221	231	241	201

Replication 4

Block 1	*Block 2*	*Block 3*	*Block 4*	*Block 5*
nqp	*nqp*	*nqp*	*nqp*	*nqp*
0-0	0-0	0-0	0-0	0-0
0-1	0-1	0-1	0-1	0-1
110	120	130	140	100
111	121	131	141	101
220	230	240	200	210
221	231	241	201	211